Guangzhou Discrete Mathematics Seminar

Hajós' Coloring Conjecture

Qiqin Xie

Fudan University, Shanghai, China

6 January 2021 (Wednesday), 8pm to 9pm
Room 416, School of Mathematics, Sun Yat-sen University
Tencent meeting ID: 320 073 530

The Four Color Theorem states that every planar graph is 4-colorable. By Kuratowski's theorem, a graph is planar if and only if it contains no K_5-subdivision or $K_{3,3}$-subdivision. The structure and the chromatic number of graphs with no $K_{3,3}$-subdivision have been studied by Wagner and Kelmans. Thus it is natural to consider the chromatic number of graphs with no K_5-subdivision.

Hajós conjectured that for any positive integer k, every graph containing no K_{k+1}-subdivision is k-colorable. However, Catlin disproved Hajós' conjecture for $k \geq 6$. Subsequently, Erdős and Fajtlowicz showed that Hajós' conjecture fails for almost all graphs. It is not hard to prove the conjecture is true for $k \leq 3$. Hajós' conjecture remains open for $k = 4, 5$. In this talk, I will survey the history of Hajós' conjecture and discuss some recent results on the problem.

This talk is based on some joint work with Shijie Xie, Xingxing Yu, and Xiaofan Yuan.